## M.A. FINAL {MATHEMATICS}

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| 1. | Let X be the real normed linear space of all real sequences with finitely many non-zero terms with supremum norm and T:X → X be a one to one and onto linear operator defined by:<br>$T(x_1, x_2, \ldots\ldots) = \left(x_1, \frac{x_2}{2^2}, \frac{x_3}{3^3}, \ldots\ldots\ldots\right)$<br>Then<br>a. T is bounded but $T^{-1}$ is not bounded<br>b. T is not bounded but $T^{-1}$ is bounded<br>c. Both T and $T^{-1}$ is bounded<br>d. Neither T nor $T^{-1}$ is bounded |
| 2. | Let X and Y be normed linear spaces and $\{T_n\}$ be a sequence of bounded linear operators from X to Y. Consider the statements<br>P: $\{\|T_n(x)\|: n \in N\}$ is bounded for each $x \in X$<br>Q: $\{\|T_n(x)\|: n \in N\}$ is bounded<br>Then<br>a. if P and Q then both X and Y are Banach spaces<br>b. if p implies Q then only one of X and Y is a Banach spaces.<br>c. if X is Banach then $P \Rightarrow Q$<br>d. if Y is Banach then $P \Rightarrow Q$ |
| 3. | Let $f: (C_{00}, \|\ \|_1) \to C$ be a non-zero continuous linear functional. The number of Hahn-Banach extensions of $f$ to $l_1$ is<br>a. One  b. Two  c. Three  d. Infinite |
| 4. | Let $\|\ \|_p$ denote the p-norm on $R^2$, $1 \le p \le \infty$. If $\|\ \|_p$ satisfies the parallelogram law, then p is equal to<br>a. One  b. Two  c. Three  d. Any value |
| 5. | Let μ be a signed measure on a measurable space (X, A). Then there exists a positive set P and a negative set Q such that $P \cap Q = \phi$, $X = P \cup Q$. This is the statement of<br>a. Radon Nikdym Theorem<br>b. Lebesgue Decomposition Theorem<br>c. Hahn Decomposition Theorem<br>d. Fubini's Theorem |
| 6. | Which of the following statements are true for a normed linear space (N, $\|\ \|$)<br>1. Every absolutely summable series in N is summable ⇔ N is a Banach space<br>2. Every convergent sequence in N is a Cauchy sequence ⇒ N is a Banach space<br>3. Every Cauchy sequence in N is convergent ⇔ N is a Banach space<br>4. Every summbale series in N is summable ⇔ N is a Banach space<br>a. (1) and (3)  b. (1) and (2)<br>c. (1) and (4)  d. (2) and (3) |
| 7. | Let T be a linear transformation of a normed linear space N into another normed linear space $N^1$. Which of the following statements are is not true<br>1. T is continuous iff there exists a real number $K \ge 0$ such that $\|T(x)\| \le K\|x\|$ for all $x \in N$ (where N is set of natural numbers)<br>2. T is continuous iff for any sequence $<x_n>$ in N converging to $x \in N$, the sequence $<T(x_n)>$ in $N^1$ converges to $T(x) \in N^1$<br>3. T is continuous iff T is bounded<br>4. None of the above is true |

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| 8. | Let x and y be any two vectors in a Hilbert space and inner product of x and y is denoted by (x, y), then Polarisation identity is given by<br>a. $\|(x,y)\| \leq \|x\|\|y\|$<br>b. $\|\|x\| - \|y\|\| \leq \|x - y\|$<br>c. $4(x, y) = \|x + y\|^2 - \|x - y\|^2 + i\|x + iy\|^2 - i\|x - iy\|^2$<br>d. $\|x + y\|^2 + \|x - y\|^2 = 2\|x\|^2 + \|y\|^2$ |
| 9. | If $\phi(n)$ is defined a sthe positive integers relative prime to n which do not exceed n, n be a positive integer then $\phi(mn) = \phi(m)\,\phi(n)$ provided<br>a. m < n<br>b. m and n are relatively prime<br>c. m > n<br>d. None of these |
| 10. | The highest power of 13 in 1500! is<br>a. 1247 b. 1260<br>c. 1237 d. 1227 |
| 11. | The congruence $x^2 \equiv a(mod\ p), (a,p) = 1$, p is prime has a solution iff<br>a. $a^{\frac{p-1}{2}} \equiv 1(mod\ p)$<br>b. $a^{\frac{p+1}{2}} \equiv 1(mod\ p)$<br>c. $a^p \equiv a(mod\ p)$<br>d. None of these |
| 12. | The congruence $a^x \equiv 1(mod\ m)$is solvable iff<br>1. a < m<br>2. a > m<br>3. a and m are relatively prime<br>4. None of these |
| 13. | The generating function of the numeric function $a_r = 1, r \geq 0$ is<br>a. $\frac{1}{1-x} for\ \|x\| < 1$<br>b. $\frac{1}{1-x} for\ \|x\| = 1$<br>c. $\frac{1}{1-x} for\ \|x\| > 1$<br>d. $\frac{1}{1-x} for$ all values of x |
| 14. | Two functions f and g are called orthogonal with respect to the weight function p on the interval $a \leq x \leq b$ iff<br>a. $\int_a^b f(x)g(x)dx = 0$<br>b. $\int_a^b e^{-p(x)} f(x)g(x)dx = 0$<br>c. $\int_a^b f(x)g(x)p(x)dx = 0$<br>d. None of these |
| 15. | If $f_n = n^p$, p is a positive integer then z- transform of $f_n$ is<br>a. $-z\frac{d}{dz}z(n^{p-1})$<br>b. $z\frac{d}{dz}z(z^p)$<br>c. $-z\frac{d}{dz}z(n^p)$<br>d. None of these |
| 16. | The equation of streem lines is<br>a. $\bar{q} \times d\bar{r} = 0$<br>b. $\bar{q}.d\bar{r} = 0$ |

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| | c. $\bar{r}.d\bar{q} = 0$<br>d. $\bar{r} \times d\bar{q} = 0$ |
| 17. | The differential equations of path lines are<br>a. $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$<br>b. $\frac{dx}{\xi} = \frac{dy}{\eta} = \frac{dz}{\varsigma}$<br>c. $\frac{dx}{dt} = u, \frac{dy}{dt} = v, \frac{dz}{dt} = w$<br>d. None of these |
| 18. | For incompressible flow, we have<br>a. $div\ \bar{q} = 0$<br>b. $\frac{d\bar{q}}{dt} = 0$<br>c. $\frac{dq}{dt} = 0$<br>d. None of these |
| 19. | The relation between potential function $\phi$ and stream function $\psi$ is<br>a. $\frac{\partial \emptyset}{\partial x} = \frac{\partial \psi}{\partial y}$ and $\frac{\partial \emptyset}{\partial y} = \frac{\partial \psi}{\partial x}$<br>b. $\frac{\partial \emptyset}{\partial x} = \frac{\partial \psi}{\partial y}$ and $\frac{\partial \emptyset}{\partial y} = -\frac{\partial \psi}{\partial x}$<br>c. $\frac{\partial \emptyset}{\partial x} = -\frac{\partial \psi}{\partial y}$ and $\frac{\partial \emptyset}{\partial y} = \frac{\partial \psi}{\partial x}$<br>d. None of these |